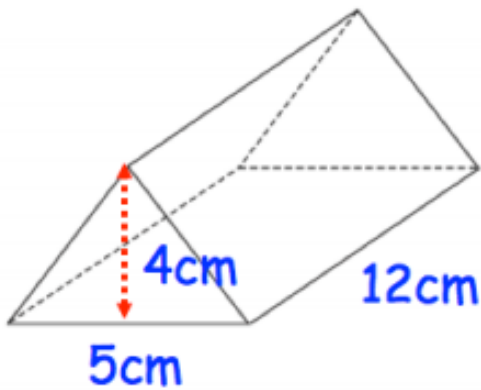


Volume

Volume of prisms

To find the volume of a prism you multiply the cross-sectional area (e.g. the cross section of a cylinder is a circle) by the depth.

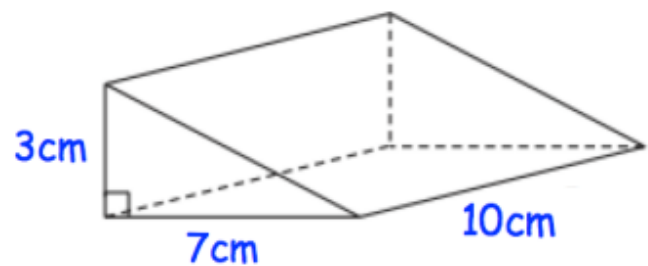
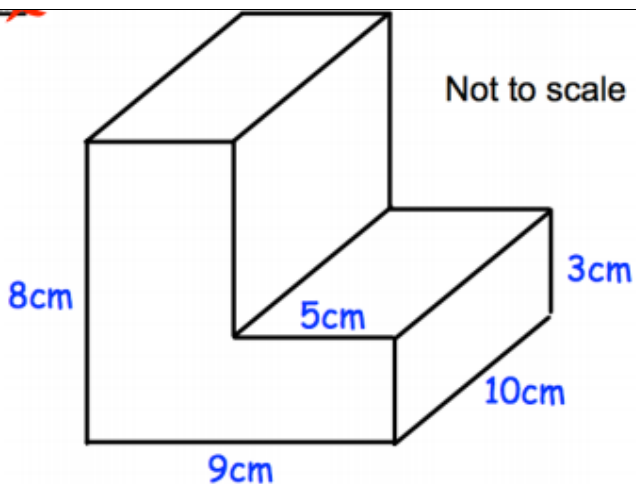
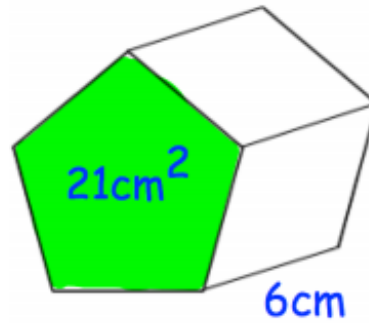
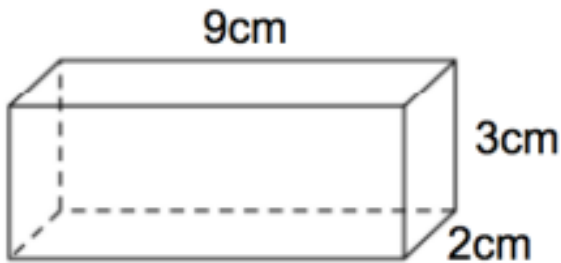
Example

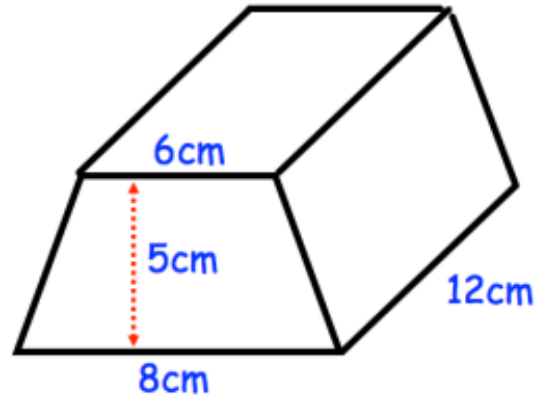


The cross-section is a triangle.

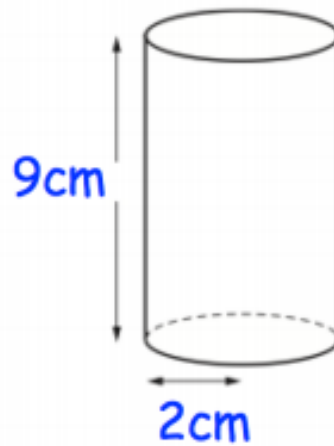
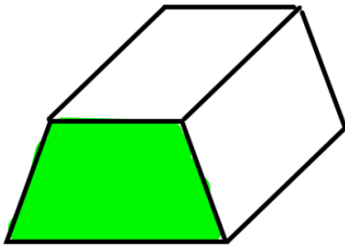
$$\begin{aligned}\text{Area of the triangle} &= \frac{1}{2} \text{ base} \times \text{height} \\ &= \frac{1}{2} (4 \times 5) = \mathbf{10\text{cm}^2}\end{aligned}$$

$$\begin{aligned}\text{Volume} &= \text{area of cross section} \times \text{depth} \\ &= 10 \times 12 = \mathbf{120\text{cm}^3}\end{aligned}$$





The area of the cross-section is 55cm^2 . The volume of the prism is 330cm^3 .
Find the length of the prism.



Find the volume. Leave your answers in terms of π .

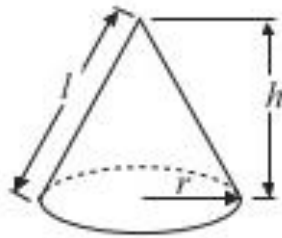
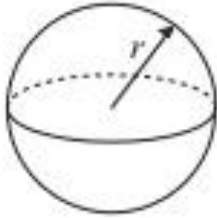
Harder volume

You are given these formulas:

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

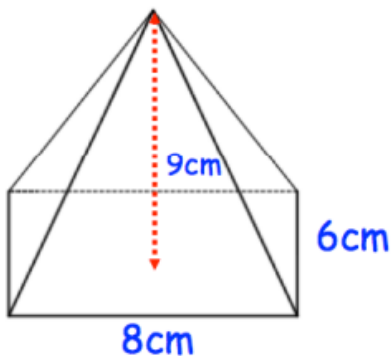
$$\text{Surface area of sphere} = 4\pi r^2 \quad \text{Curved surface area of cone} = \pi r l$$



You need to remember:

Volume of a pyramid = $\frac{1}{3}$ area of base x height

Examples



Volume = $\frac{1}{3}$ area of base x height

$$= \frac{1}{3} (8 \times 6) \times 9$$

$$= \frac{1}{3} (48) \times 9$$

$$= 144\text{cm}^3$$

The volume of the cone is 150cm^3 . Find the height of the cone to one decimal place.

Volume of cone = $\frac{1}{3} \pi r^2 h$

Substitute the values you know into the formula

$$150\text{cm}^3 = \frac{1}{3} \times \pi \times 4^2 \times h$$

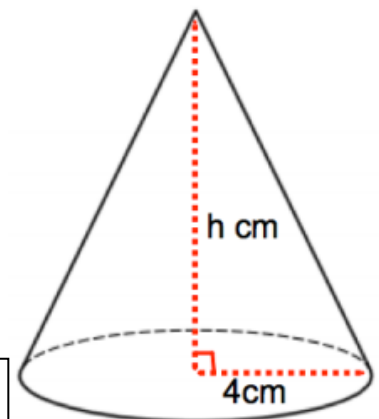
$$150\text{cm}^3 = \frac{1}{3} \times \pi \times 16 \times h$$

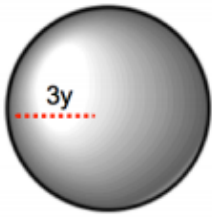
$$150\text{cm}^3 = 16.755 \times h$$

$$150\text{cm}^3 \div 16.755 = h$$

$$\mathbf{9.0\text{cm} = h}$$

Simplify by multiplying the numbers





The sphere and cone have the same volume. Find an expression for h in terms of y .
(6 marks)

$$\text{Volume sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \pi \times (3y)^3$$

$$= \frac{4}{3} \times \pi \times 27y^3$$

$$= \mathbf{36\pi y^3}$$

$$\text{Volume cone} = \frac{1}{3} \pi r^2 h$$

Substitute the values into each formula and simplify

$$= \frac{1}{3} \times \pi \times (3y)^2 \times h$$

$$= \frac{1}{3} \times \pi \times 9y^2 \times h$$

$$= \mathbf{3\pi y^2 h}$$

Volume of sphere = Volume of cone

$$36\pi y^3 = 3\pi y^2 h$$

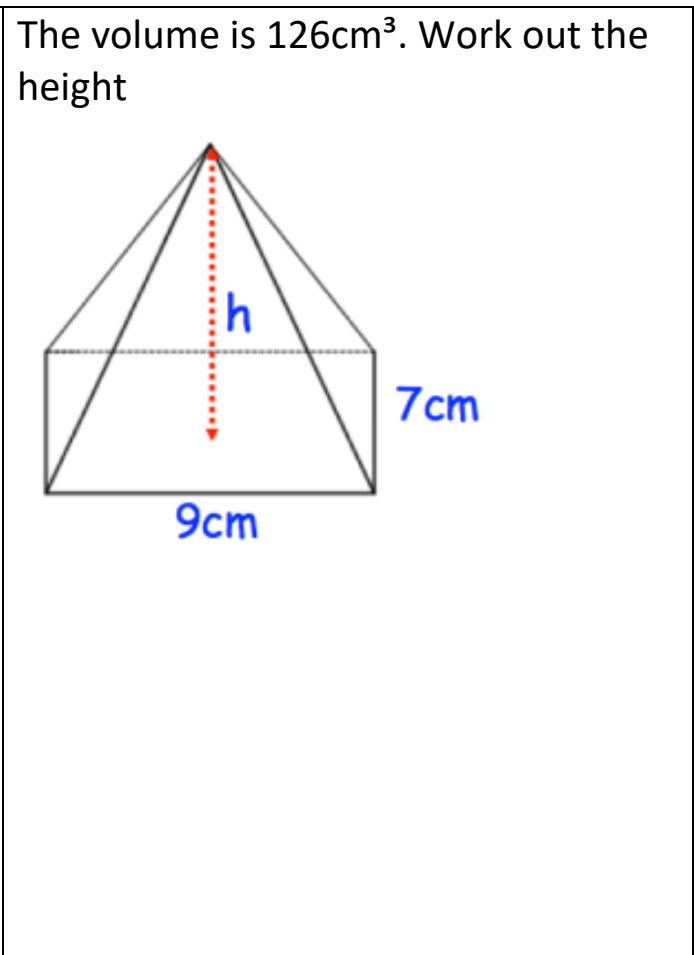
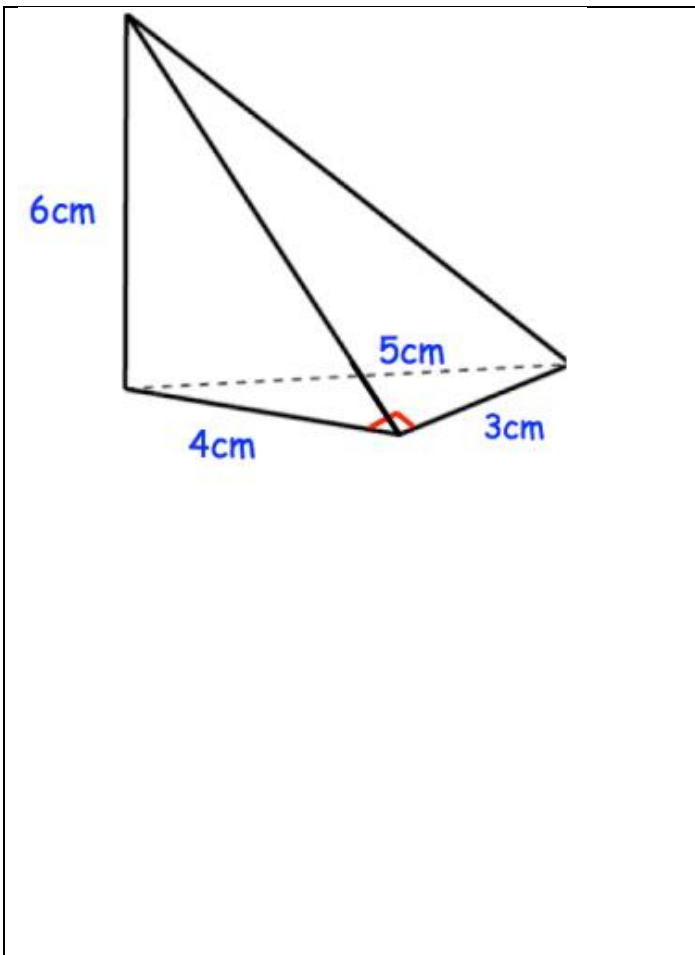
$$\div \pi \qquad \qquad \div \pi$$

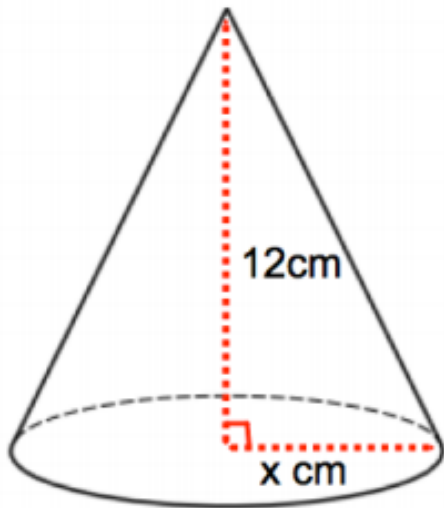
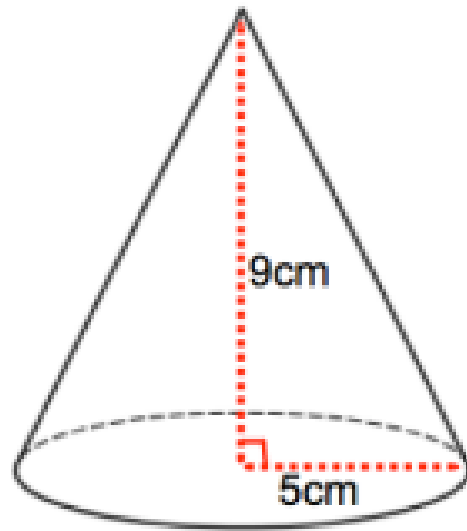
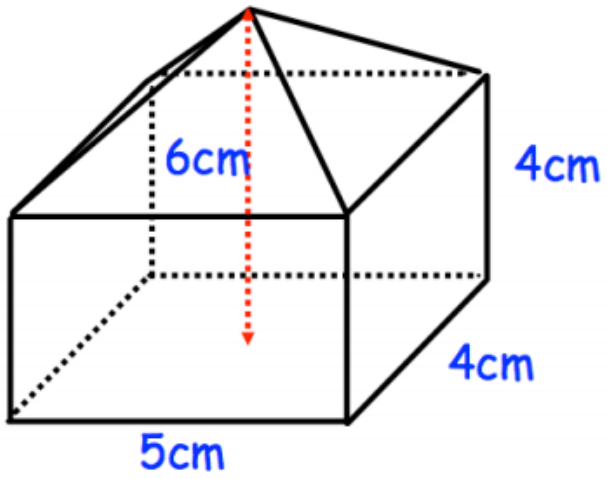
$$36y^3 = 3y^2 h$$

$$\div 3y^2 \qquad \qquad \div 3y^2$$

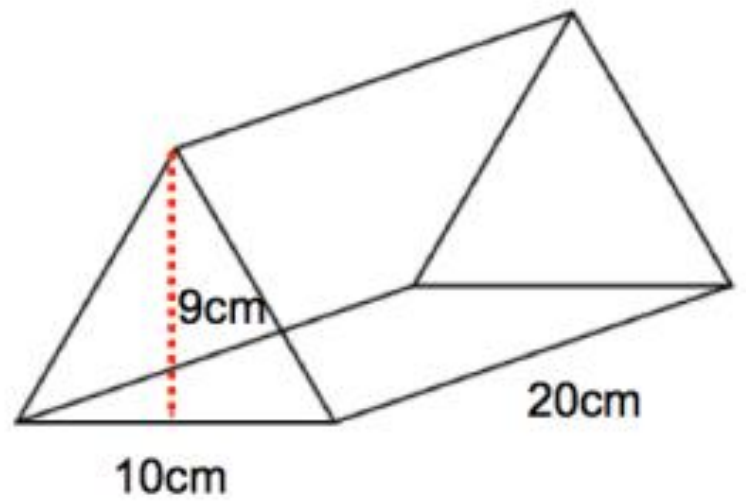
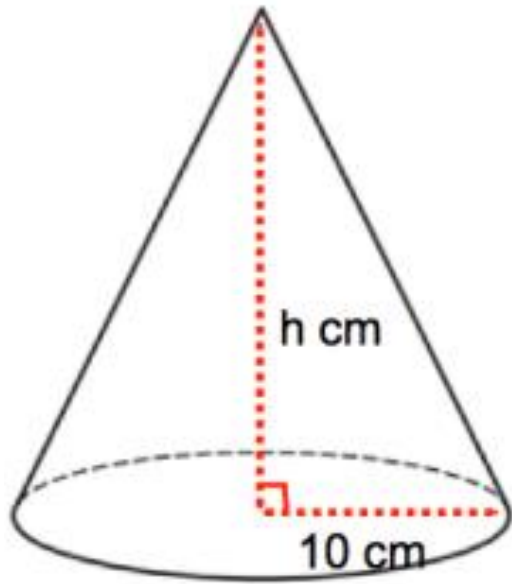
$$\mathbf{12y = h}$$

We know that the volumes are equal. We then rearrange to form an expression with $h =$





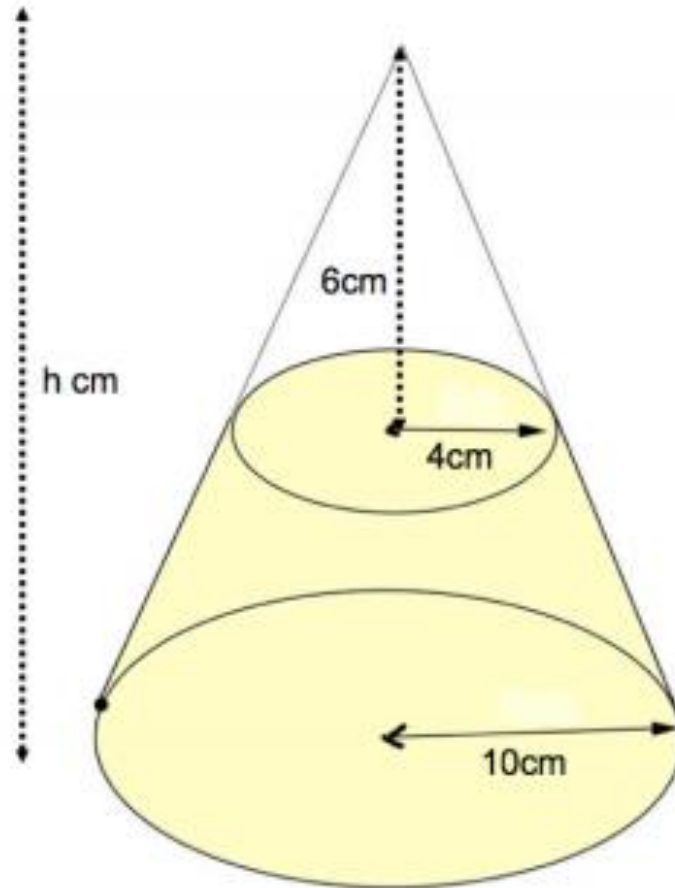
The volume is 170 cm^3 . Calculate the radius to 1 decimal place.



Both solids have the same volume.

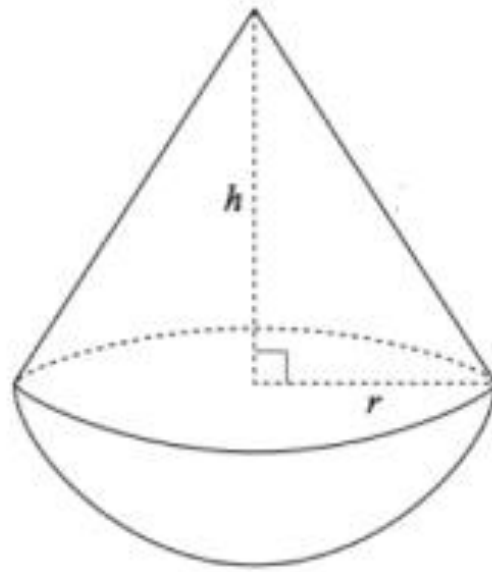
Calculate the height of the cone.

A cone below has base radius 10cm and height h cm.
A smaller cone radius 4cm and height 6cm is cut from the top.
The frustum is shown below.



Calculate the volume of the frustum.

The diagram shows a solid made up of a cone and a hemisphere.



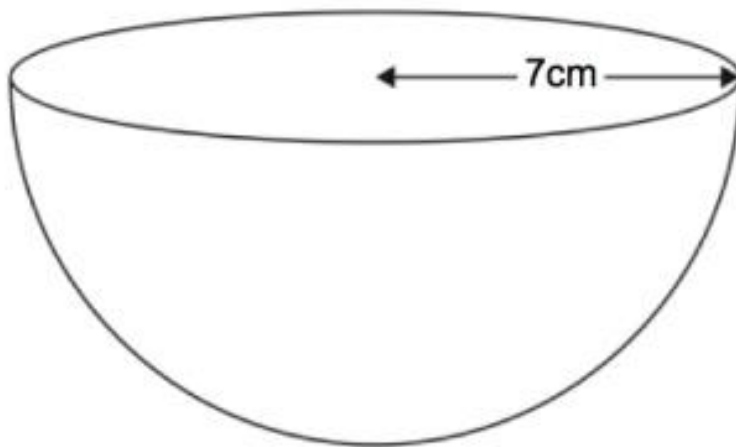
The radius of the cone is 5cm.

The height of the cone is 8cm.

The volume of a hemisphere is $\frac{2}{3}\pi r^3$

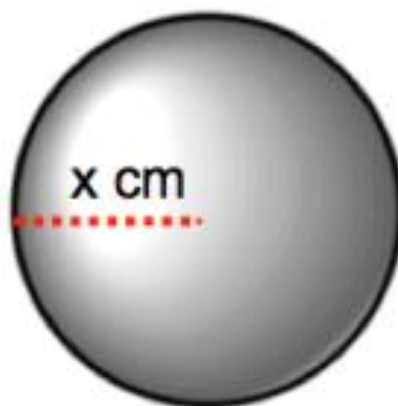
Calculate the volume of the solid.

Shown below is a hemisphere.



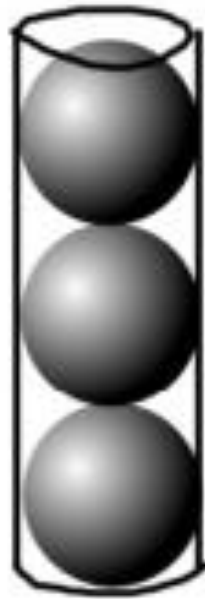
Calculate the volume of the hemisphere.

A sphere has volume 500cm^3 .



Calculate the radius of the sphere, x .

Three spheres of radius 4cm fit inside a tube.



Calculate the percentage of the tube that is not filled.

Mixed Exam Questions

Q1. The diagram shows a swimming pool in the shape of a prism.

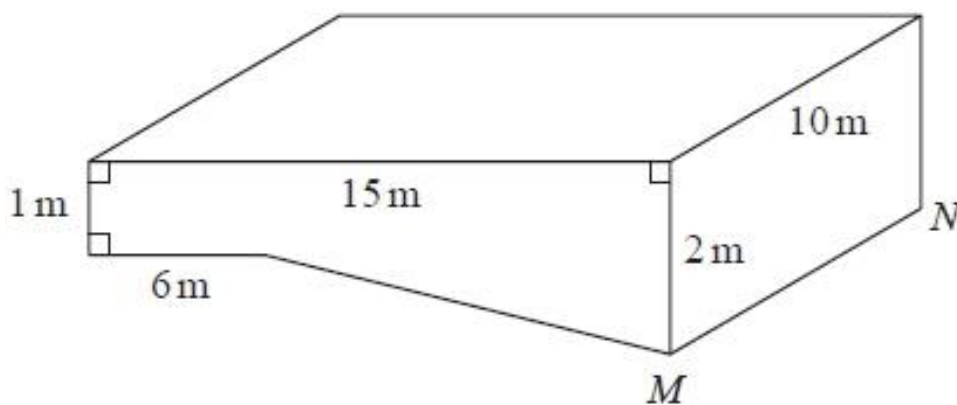


Diagram **NOT**
accurately drawn

The swimming pool is empty.

The swimming pool is filled with water at a constant rate of 50 litres per minute.

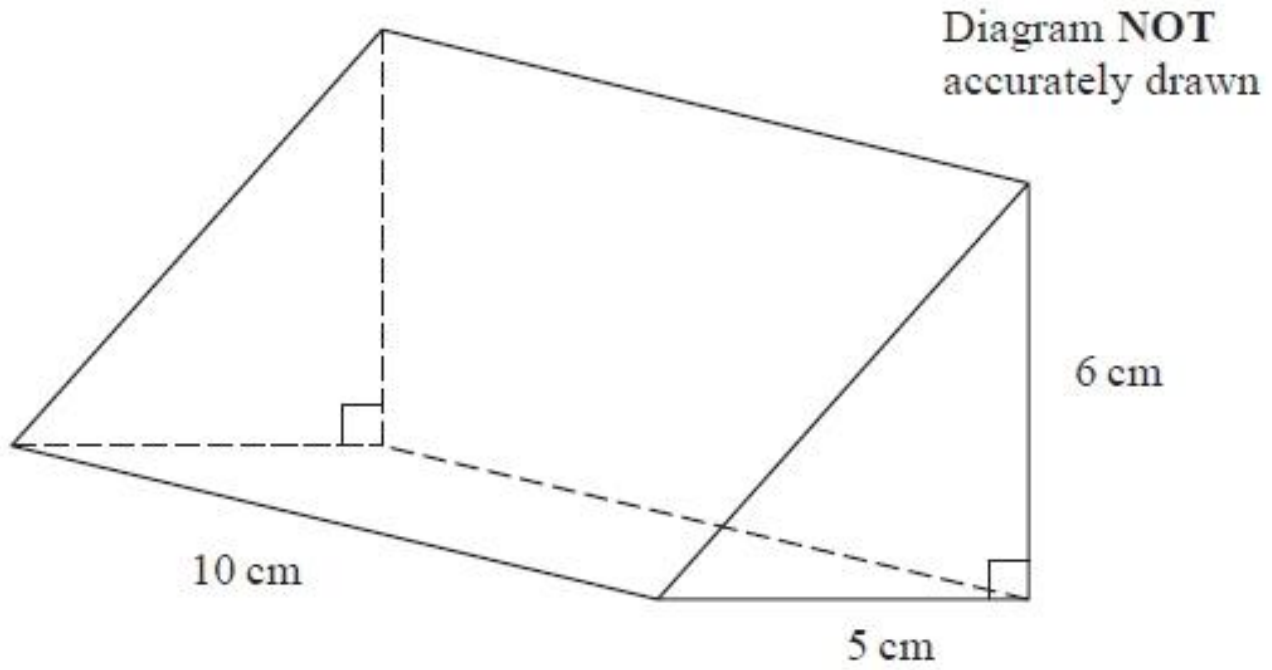
(a) Work out how long it will take for the swimming pool to be completely full of water.

Give your answer in hours.

(1 m³ = 1000 litres)

..... hours
(Total for question = 5 marks)

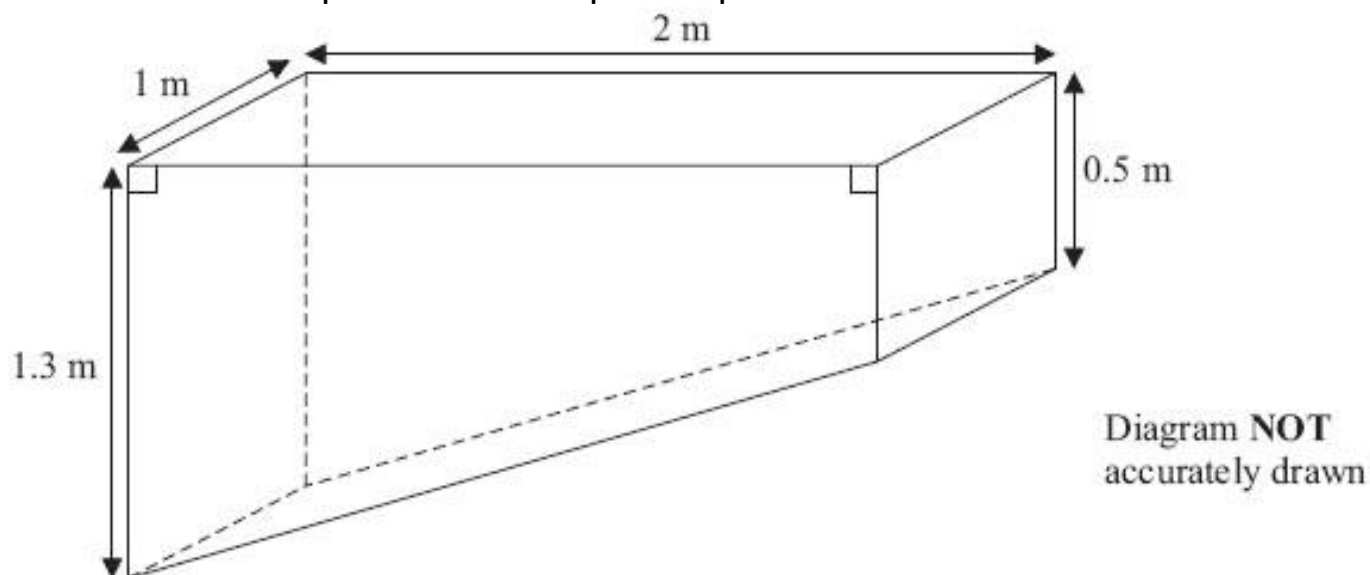
Q2. The diagram shows a triangular prism.



Work out the volume of the prism.

.....
(Total for question = 3 marks)

Q3. Sumeet has a pond in the shape of a prism.



The pond is completely full of water.

Sumeet wants to empty the pond so he can clean it.

Sumeet uses a pump to empty the pond.

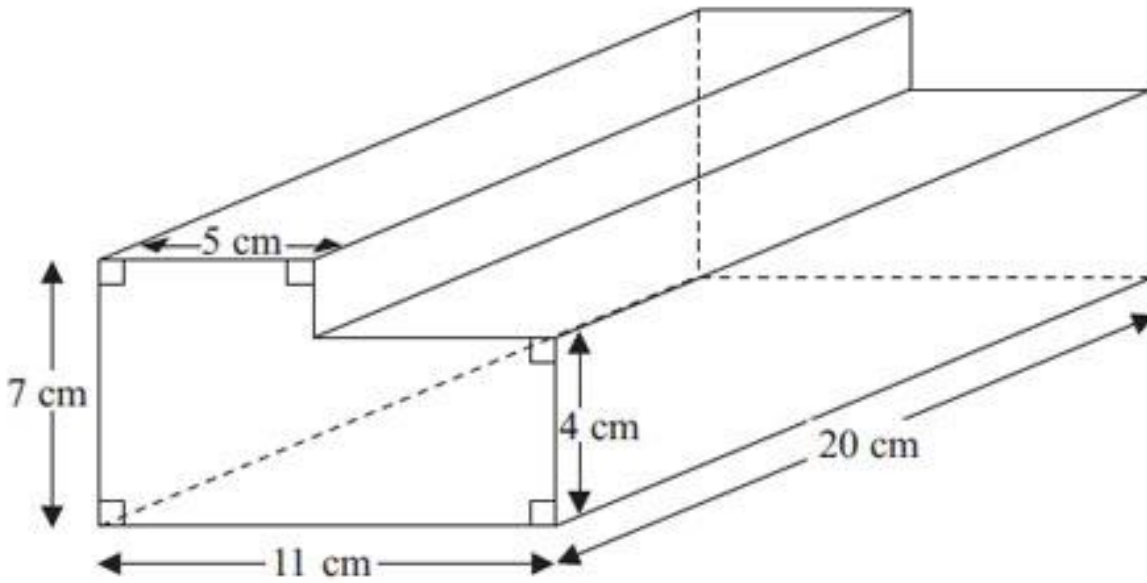
The volume of water in the pond decreases at a constant rate.

The level of the water in the pond goes down by 20 cm in the first 30 minutes.

Work out how much more time Sumeet has to wait for the pump to empty the pond completely.

.....
(Total for Question is 6 marks)

Q4. Here is a solid prism.



Work out the volume of the prism.

.....cm³
(Total for Question is 3 marks)

Q5.

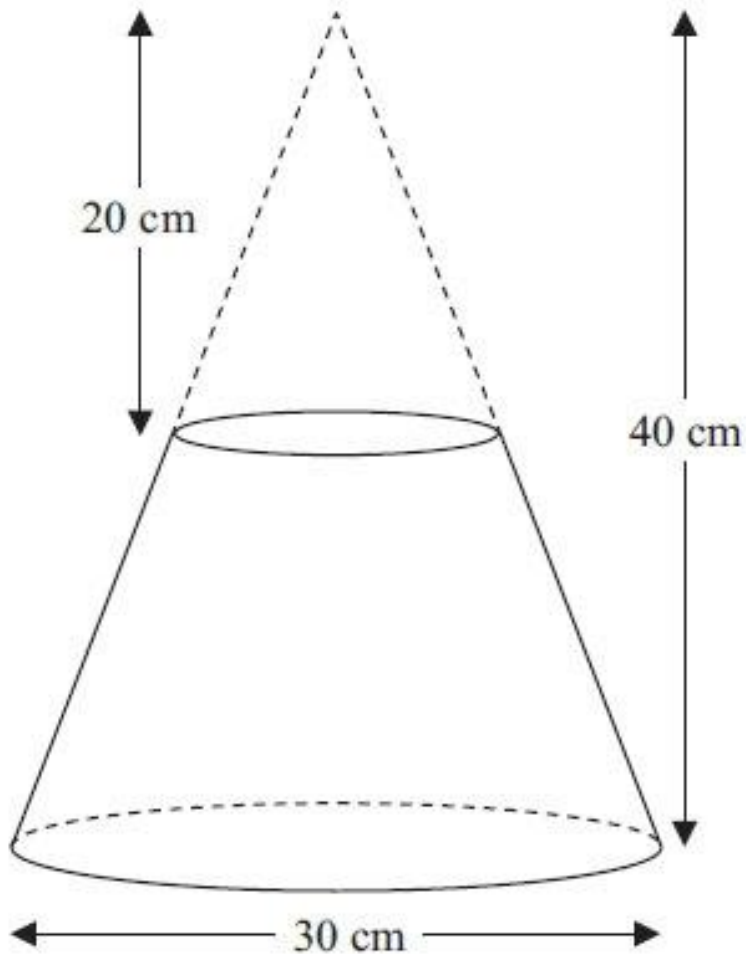


Diagram **NOT**
accurately drawn

A frustum is made by removing a small cone from a similar large cone.

The height of the small cone is 20 cm.

The height of the large cone is 40 cm.

The diameter of the base of the large cone is 30 cm.

Work out the volume of the frustum.

Give your answer correct to 3 significant figures.

.....cm³
(Total for Question is 4 marks)

Q6. The diagram shows a solid made from a hemisphere and a cone.

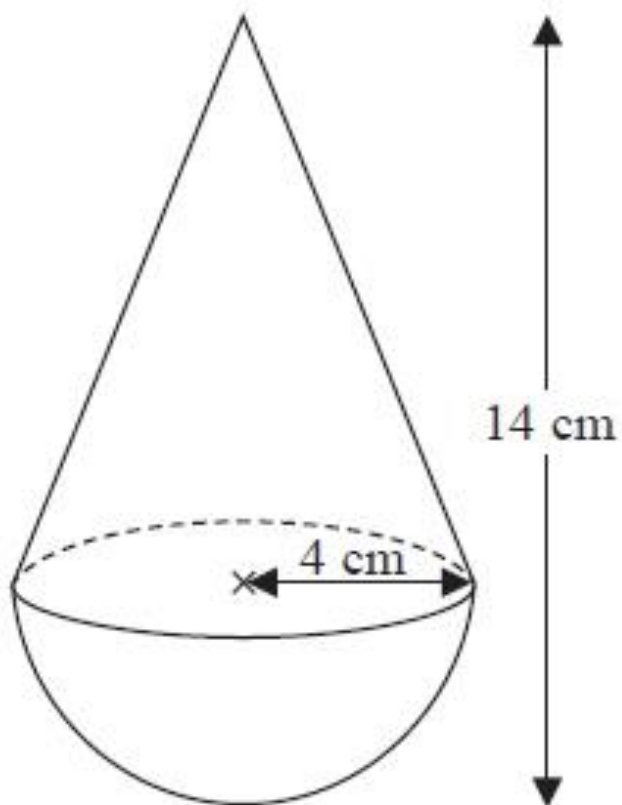


Diagram NOT accurately drawn

The radius of the hemisphere is 4 cm.

The radius of the base of the cone is 4 cm.

Calculate the volume of the solid.

Give your answer correct to 3 significant figures.

.....cm³

(Total for Question is 3 marks)

Q7.

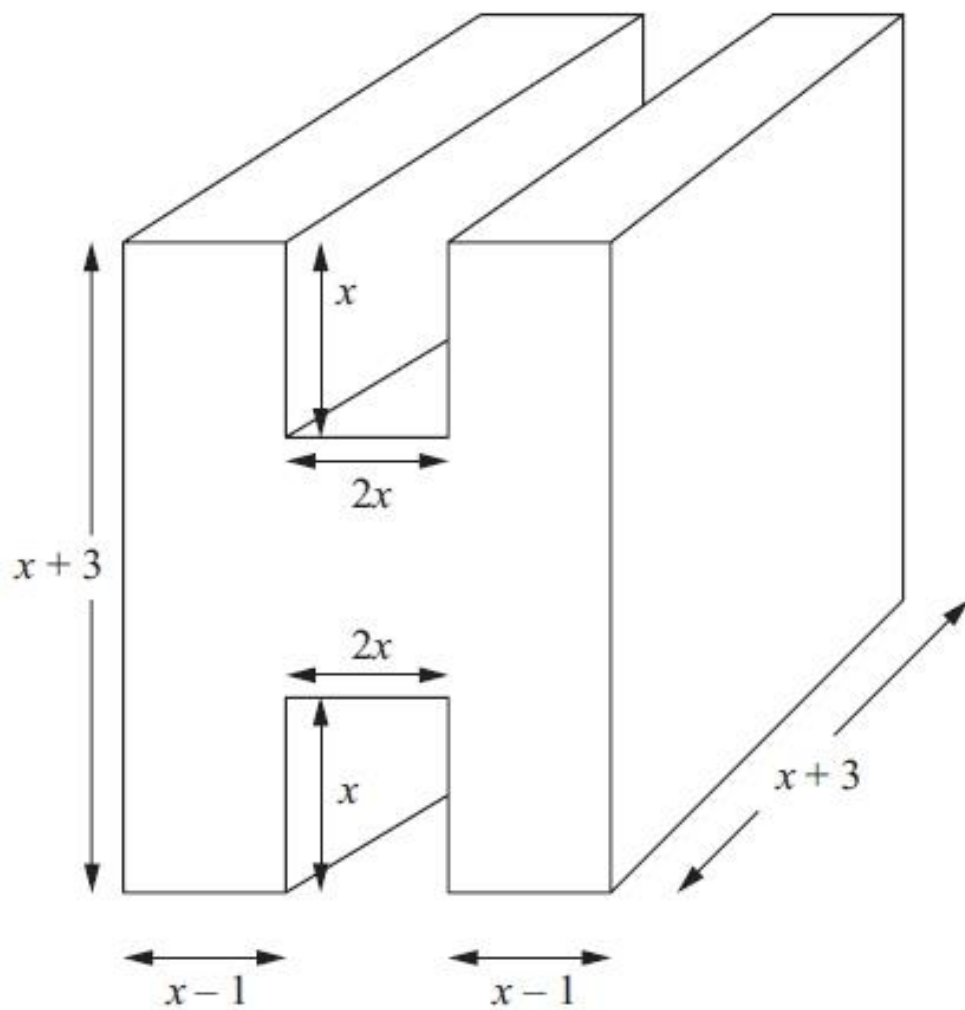


Diagram NOT

accurately drawn

The diagram shows a prism.

All measurements are in cm.

All corners are right angles.

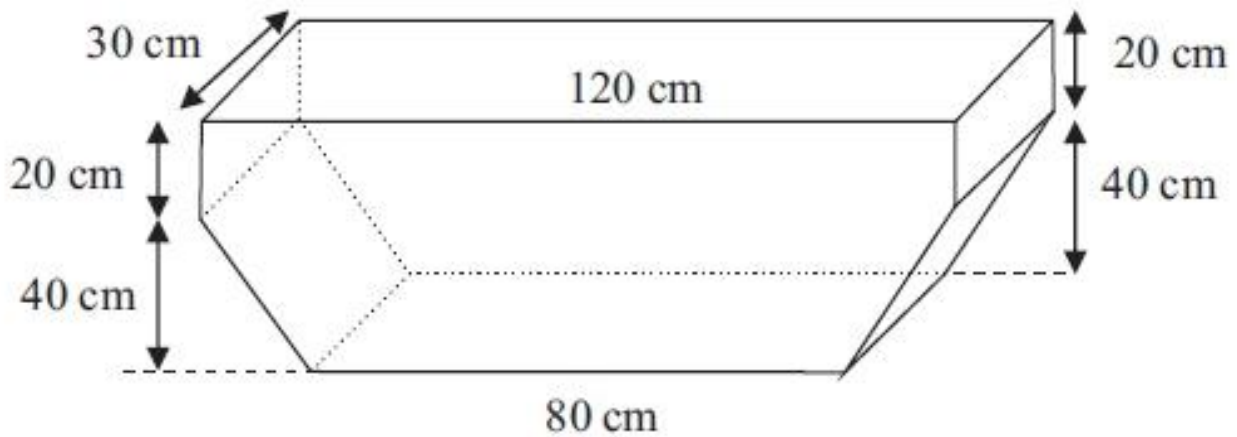
The volume of the prism is $V \text{ cm}^3$.

Find a formula for V .

$V = \dots\dots\dots$
(Total for Question is 4 marks)

Q8.

A water trough is in the shape of a prism.



Hamish fills the trough completely.

Water leaks from the bottom of the trough at a constant rate.

2 hours later, the level of the water has fallen by 20 cm.

Water continues to leak from the trough at the same rate.

How many more minutes will it take for the trough to empty completely?

..... minutes
(Total for Question is 6 marks)

Q9.

Here is a vase in the shape of a cylinder.

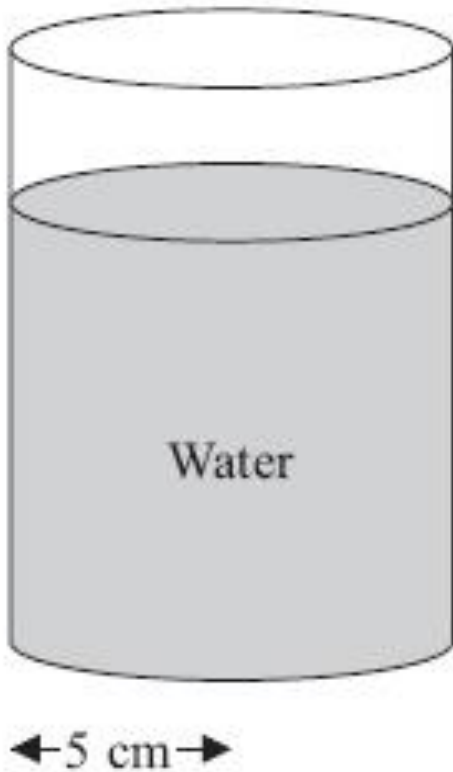


Diagram **NOT**
accurately drawn

The vase has a radius of 5 cm.

There are 1000 cm^3 of water in the vase.

Work out the depth of the water in the vase.

Give your answer correct to 1 decimal place.

(Total for Question is 3 marks)

Q10. The diagram shows a pyramid.

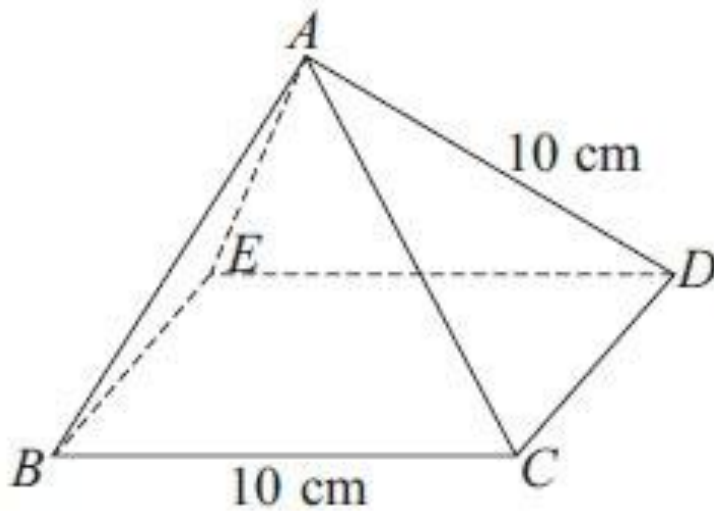


Diagram **NOT**
accurately drawn

$BCDE$ is a square with sides of length 10 cm.

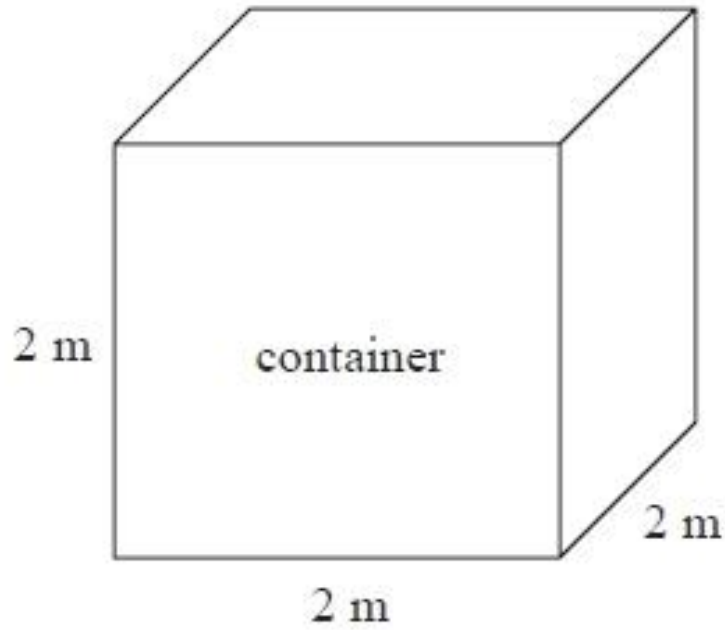
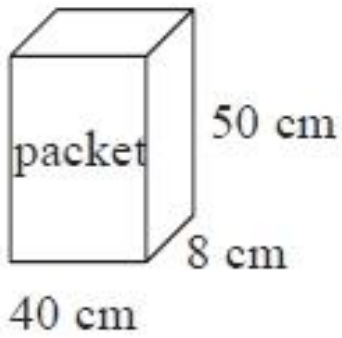
The other faces of the pyramid are equilateral triangles with sides of length 10 cm.

(a) Calculate the volume of the pyramid.

Give your answer correct to 3 significant figures.

..... cm^3
(Total for Question is 4 marks)

Q11. Ali has some packets.



Each packet has dimensions 40 cm by 8 cm by 50 cm.
Ali fills a container with these packets.
The container is a cube of side 2 m.
Ali fills the container completely with these packets.
Work out the number of packets.

.....
(Total for Question is 4 marks)

Q12. The diagram shows a prism.

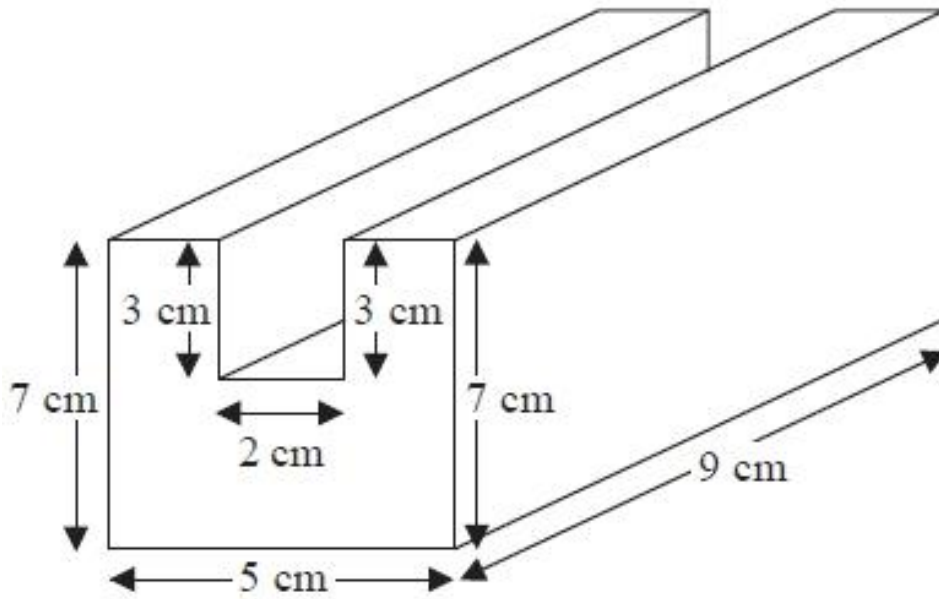


Diagram **NOT** accurately drawn

All the corners are right angles.
Work out the volume of the prism.

.....
(Total for question = 4 marks)

Q13. The diagram shows a container used to store oil.

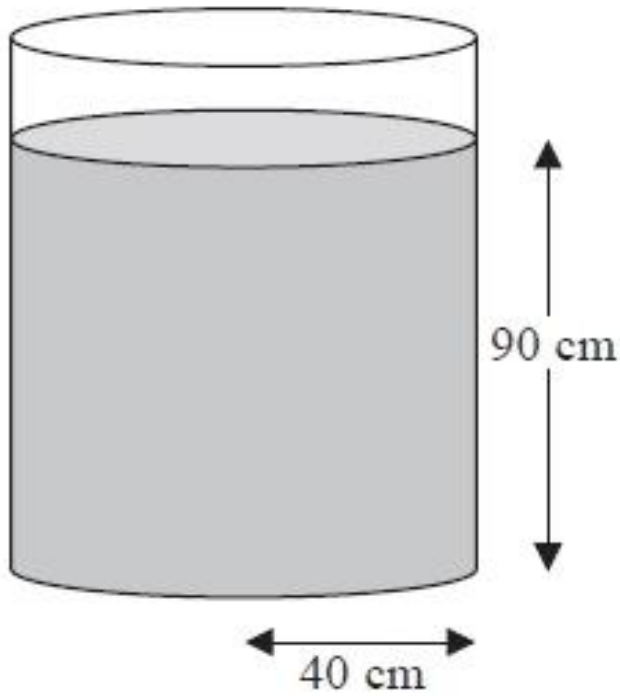


Diagram **NOT**
accurately drawn

The container is in the shape of a cylinder of radius 40 cm.

The height of the oil in the container is 90 cm.

65 litres of oil are taken from the container.

1 litre = 1000 cm^3 .

Work out the new height of the oil in the container.

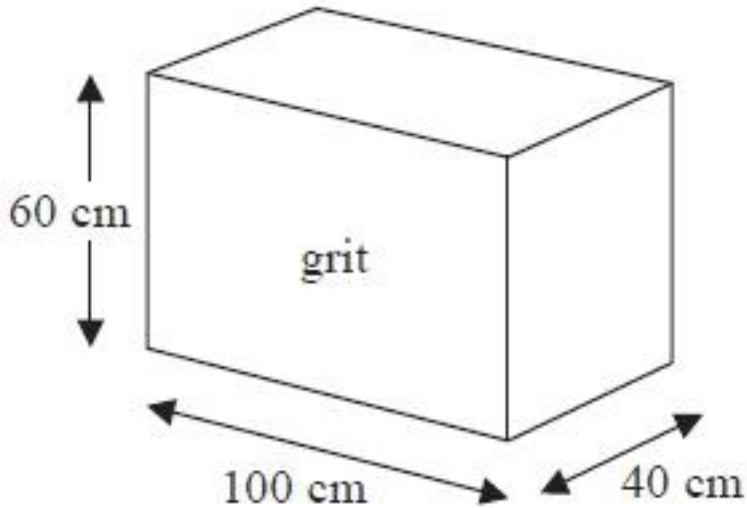
Give your answer correct to one decimal place.

.....cm

(Total for Question is 4 marks)

Q14. * The diagram shows a box for winter grit.
The box is in the shape of a cuboid.
The box is empty.

Diagram **NOT**
accurately drawn



Jon wants to fill the box with grit.
A bag of grit costs £2.50
There are 8000 cm^3 of grit in a bag.
Jon has £70 to spend on the grit.
Does Jon have enough money to buy all the grit he needs to fill the box completely?

(Total for question = 5 marks)

Examiner's Report Key Points

- Think carefully which shape is the cross-section
- A common mistake is to find the surface area- remember that the volume is the space inside the shape
- Don't forget to include the correct units e.g. cm^3
- Make sure that your **working out is clear**- you are more likely to gain marks if your calculations are well set out so that the examiner know what you are attempting to calculate.
- If the shape has a compound cross section e.g. an L-shaped prism, make sure that you write down any missing lengths that you have calculated. If you make a mistake on one of the first steps you can still get marks for doing the right calculations later on.
- Even if you are not sure what to do next, always substitute any values that you have into the formula for the volume, as this will be worth a mark.
- Carefully copy the formulas for a sphere or cone from the formula sheet if you need them in a question. Lots of people loose marks because they have made mistakes in copying the formula.
- Round your answers as late as you can in the calculation- you may lose an accuracy mark if you round too early.
- If volumes involve algebra, don't forget to use brackets when multiplying expressions e.g. $(x + 1) \times 5$ rather than $x + 1 \times 5$

Answers

Volume of prisms

54cm^3	126cm^3
470cm^3	105cm^3
423.3cm^3	420cm^3
6 cm	$36\pi\text{cm}^3$

Harder volue

12cm^3 6cm
 $93 \frac{1}{3}\text{cm}^3$ 235.62cm^3
3.7cm
8.6cm
 $468\pi\text{cm}^3$ or 1470.265cm^3
 471.2389cm^3

718.4cm³

4.924cm

Volume of cylinder = 1206.37 volume of spheres = 804.25 **33.3%**

Mixed exam questions Mark Scheme

Q1.

Answer	Mark	Notes
65	5	M1 for splitting up the cross section into separate areas and a method to find the area of one part OR for splitting up the pool into smaller prisms and a method to find the volume of one small prism, e.g. a <u>cuboid</u> M1 (dep) for a complete method to find the area of the cross section [with correct dimensions] OR for a method to find the total volume of more than one correct prism M1 (dep) for a complete method to find the volume of the pool [with correct dimensions] (= 195) M1 for "195" × 1000 ÷ 50 (=3900) oe where "195" comes from a volume A1 cao

Q2.

Answer	Mark	Notes
150 cm ³	3	M1 for a complete method to find the volume A1 for 150 B1 (indep) for cm ³

Q3.

Answer	Mark	Notes
1 hour 45 mins	6	<p>M1 for method to find volume of pond, eg $\frac{1}{2} (1.3 + 0.5) \times 2 \times 1 (= 1.8)$</p> <p>M1 for method to find the volume of water emptied in 30 minutes, eg $1 \times 2 \times 0.2 (= 0.4)$, $100 \times 200 \times 20 (= 400000)$</p> <p>A1 for correct rate, eg $0.8 \text{ m}^3/\text{hr}$, 0.4 m^3 in 30 minutes</p> <p>M1 for correct method to find total time taken to empty the pond, eg $"1.8" \div "0.8"$</p> <p>M1 for method to find extra time, eg 2 hrs 15 minutes – 30 minutes</p> <p>A1 for 1.75 hours, $1\frac{3}{4}$ hours, 1 hour 45 mins or 105 mins</p> <p>OR</p> <p>M1 for method to find volume of water emptied in 30 minutes, eg. $1 \times 2 \times 0.2 (= 0.4)$, $100 \times 200 \times 20 (= 400000)$</p> <p>M1 for method to work out rate of water loss eg. $"0.4" \times 2$</p> <p>A1 for correct rate, eg $0.8 \text{ m}^3/\text{hr}$</p> <p>M1 for correct method to work out remaining volume of water eg. $\frac{1}{2} (1.1 + 0.3) \times 2 \times 1 (= 1.4)$</p> <p>M1 for method to work out time, eg $"1.4" \div "0.8"$</p> <p>A1 for 1.75 hours, $1\frac{3}{4}$ hours, 1 hour 45 mins or 105 mins</p> <p>NB working could be in 3D or in 2D and in metres or cm throughout</p>

Q4.

Answer	Mark	Notes
1180	3	<p>M1 for a correct method to find the area of the cross section</p> <p>M1 (dep) for a complete correct method for the volume of the prism</p> <p>A1 cao</p> <p>OR</p> <p>M1 for a correct method to find the volume of one cuboid</p> <p>M1 (dep) for a complete correct method for the volume of the prism</p> <p>A1 cao</p>

Q5.

Answer	Mark	Notes
8250	4	<p>B1 for 15cm as diameter or 7.5 cm as radius of smaller cone (may be marked on diagram or used in a formula)</p> <p>M1 for a numerical expression for the volume of one cone eg. $\frac{1}{3} \times \pi \times 15^2 \times 40$ (=9424...) or $\frac{1}{3} \times \pi \times 7.5^2 \times 20$ (=1178...)</p> <p>M1 for $\frac{1}{3} \times \pi \times 15^2 \times 40$ oe $-\frac{1}{3} \times \pi \times 7.5^2 \times 20$ oe</p> <p>A1 for answer in the range 8240 – 8250</p> <p>OR</p> <p>B1 for 2^3</p> <p>M1 for a numerical expression for the volume of the large cone eg. $\frac{1}{3} \times \pi \times 15^2 \times 40$ (=9424...)</p> <p>M1 volume of frustrum = $\frac{7}{8} \times \frac{1}{3} \times \pi \times 15^2 \times 40$ oe</p> <p>A1 for answer in the range 8240 – 8250</p>

Q6.

Answer	Mark	Notes
302	3	<p>M1 for $\frac{1}{2} \times \frac{4}{3} \times \pi \times 4^3$ oe (= 133.9 – 134.2)</p> <p>M1 for $\frac{1}{3} \times \pi \times 4^2 \times 10$ oe (= 167.4 – 167.7)</p> <p>A1 for 301 – 302 (or 96π or $\frac{288}{3}\pi$)</p>

Q7.

Working	Answer	Mark	Notes
Unknown length = $x + 3 - x - x = 3 - x$ Cross-sectional area $= (x + 3)(x - 1) + (x + 3)(x - 1) + (3 - x)(2x)$ $= x^2 + 2x - 3 + x^2 + 2x - 3 + 6x - 2x^2$ $= 4x - 6 + 6x$ $= 10x - 6$ Volume $= (10x - 6)(x + 3)$ $= 10x^2 + 24x - 18$ OR Unknown length = $x + 3 - x - x = 3 - x$ Volume $= (x + 3)(x + 3)(x - 1) + (x + 3)(x + 3)(x - 1) + (2x)(3 - x)(x + 3)$ $= (10x - 6)(x + 3)$ $= 10x^2 + 24x - 18$	$10x^2 + 24x - 18$	4	B1 for $x + 3 - x - x$ oe or $3 - x$ seen or $x - 1 + 2x + x - 1$ oe or $4x - 2$ seen M1 for correct expression for 1 area from cross-section or for 1 volume of cuboid(s) (brackets not needed) M1 for correct method for total cross-sectional area OR at least 2 volumes added OR volume of surrounding cuboid – at least 1 vol (brackets needed) A1 for $10x^2 + 24x - 18$ oe

Q8.

Working	Answer	Mark	Notes
45	200 minutes	6	M1 for $120 \times 20 \times 30 (= 72000)$ M1 for "72000" ÷ 120 A1 for 600cm^3 min oe M1 for $\frac{1}{2} \times (120 + 80) \times 40 \times 30 (= 120000)$ M1 for "120000" ÷ "600" A1 for 200 minutes or 3 hours 20 mins oe SC B1 for 4 hours

Q9.

Answer	Mark	Notes
12.7	3	M1 for $3.142 \times 5 \times 5$ oe or $3.142 \times 5 \times 5 \times 'h'$ (=78.5 – 78.55) M1 for $1000 \div (3.142 \times 5 \times 5)$ A1 for 12.7 – 12.8 NB: multiples of π acceptable for M marks

Q10.

Working	Answer	Mark	Notes
<p>Let O be the centre of the base. $OB^2 + OC^2 = 10^2$; $OB^2 = 50$ $AO^2 = AB^2 - OB^2 = 50$ $Vol = \frac{1}{3} \times 10^2 \times \sqrt{50}$</p> <p>OR Let M be the midpt of side BC and let O be the centre of the base. $AM^2 + MC^2 = 10^2$; $AM^2 = 75$ $AO^2 = AM^2 - MO^2 = 50$ $Vol = \frac{1}{3} \times 10^2 \times \sqrt{50}$</p>	236	4	<p>M1 correct method to start to find BD or BO using triangle OBC or triangle BCD (oe) Eg. $OB^2 + OC^2 = 10^2$ or $BO^2 = 50$ or $BO = \sqrt{50}$ (=7.07..) or $BO = \frac{\sqrt{200}}{2}$ or $10^2 + 10^2 = BD^2$ or $BD^2 = 200$ or $BD = \sqrt{200}$ (=14.1..) M1 (dep) correct method to find height of pyramid using triangle AOB Eg. $AO^2 = 10^2 - \sqrt{50}^2$ or $AO^2 = 50$ or $AO = \sqrt{50}$ (=7.07..) M1 (indep) $\frac{1}{3} \times 10^2 \times \sqrt{50}$ (but not $\frac{1}{3} \times 10^2 \times 10$) A1 235 – 236</p>

Q11.

Answer	Mark	Notes
500	4	<p>M1 for a correct method to convert cm to m or m to cm or cm^3 to m^3 or m^3 to cm^3 (can be implied eg 4 packets drawn in container height)</p> <p>M1 for correct method for one volume or correct method to get at least 2 multipliers from packet to container (can be implied on the diagram)</p> <p>M1 for complete correct method (ignore incorrect conversions)</p> <p>A1 cao</p>

Q12.

Answer	Mark	Notes
261 cm ³	4	M1 for complete method to find the area of cross section or to find the volume of a cuboid of depth 9 M1 for complete method to find the volume of the prism eg $(5 \times 7 - 2 \times 3) \times 9$ or $(35 - 6) \times 9$ A1 for 261 B1 (indep) for cm ³

Q13.

Answer	Mark	Notes
77 to 77.2	4	M1 for $\pi \times 40^2 \times 90$ (= 452389. ...) M1 for "452389. ..." - 65000 (= 387389. ...) M1 (dep on at least M1) for "387389. ..." \div ($\pi \times 40^2$) A1 for answer in the range 77 to 77.2 OR M1 for $\pi \times 40^2$ (= 5026. ...) M1 for $65000 \div$ "5026." (= 12.93...) M1 (dep on at least M1) for $90 -$ "12.93..." A1 for answer in the range 77 to 77.2

Q14.

Answer	Mark	Notes
NO figures and comparisons	5	M1 for $100 \times 40 \times 60$ (= 240 000) M1 for "240 000" \div 8000 (= 30) M1 for "30" \times 2.50 (= 75) A1 for 240 000 and 75 C1 (dep on M1) for comparing the cost of grit with £70 ft from their working OR M1 for $70 \div 2.50$ (= 28) M1 for "28" \times 8000 (= 224 000) M1 for $100 \times 40 \times 60$ (= 240 000) A1 for 240 000 and 224 000 C1 (dep on M1) for comparing values of grit needed with that which can be bought for £70 ft from their working